Name: Date: Period:

Proving that a Quadrilateral is a Parallelogram

Lesson Objective IWBAT prove that a quad. is a //-gram.

You can only prove that a quadrilateral is a parallelogram using its definition (below):

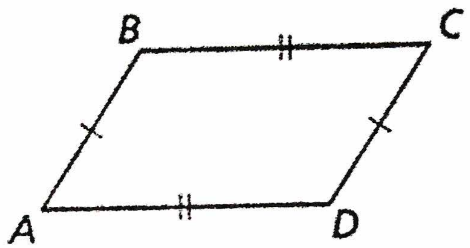
2 pairs of opp. // sides

However, there are theorems that allow us to use other properties of quadrilaterals to conclude that it is a parallelograms. They are as follows:

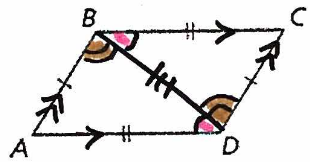
Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent,

then the quadrilateral is a parallelogram.



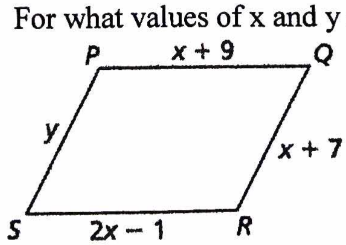
PROOF: **Given**: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
Prove: ABCD is a parallelogram



Statements	Reasons
① $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$	① given
② Draw $\overline{BD}$	② Thru 2 pts. is 1 line
③ $\overline{BD} \cong \overline{DB}$	③ reflex. prop. $\cong$
④ $\triangle ADB \cong \triangle CBD$	④ SSS $\cong$
⑤ $\angle CBD \cong \angle ADB$	⑤ CPCTC
$\angle ABD \cong \angle CDB$	
⑥ $\overline{BC} \parallel \overline{DA}$, $\overline{AB} \parallel \overline{CD}$	⑥ alt. int. ∠s $\cong \rightarrow \parallel$
⑦ ABCD is a //-gram	⑦ def. of a //-gram.

Example:

1. For what values of x and y is quadrilateral PQRS a parallelogram?



$x+9=2x-1$

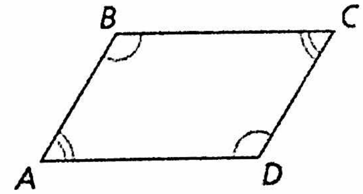
$\boxed{x=10}$

$\boxed{y=17}$

Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent,

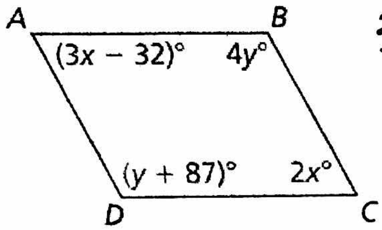
then the quadrilateral is a parallelogram.



PROOF: end of class (if time permits). Will be on classroom website tonight if you are so curious to look it up ☺

Example:

2. For what values of x and y is quadrilateral ABCD a parallelogram?



$3x - 32 = 2x$

$\boxed{x = 32}$

$4y = 87 + y$

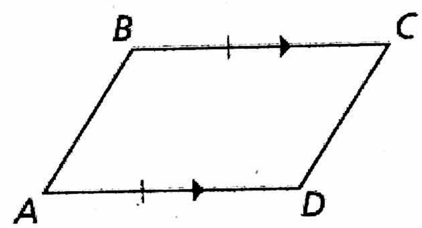
$3y = 87$

$\boxed{y = 29}$

Opposite Sides Parallel and Congruent Theorem

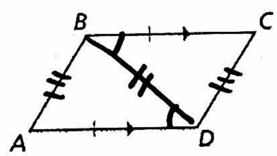
If one pair of opposite sides of a quadrilateral are congruent and parallel,

then the quadrilateral is a parallelogram.



PROOF: **Given:** $\overline{BC} \cong \overline{AD}$ and $\overline{BC} \parallel \overline{AD}$

Prove: ABCD is a parallelogram



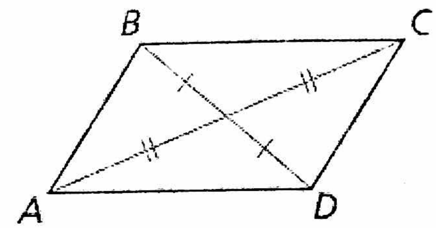
Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements	Reasons
① $\overline{BC} \cong \overline{AD}$, $\overline{BC} \parallel \overline{AD}$	① given
② Draw $\overline{BD}$	② thru 2 pts. is 1 line
③ $\overline{BD} \cong \overline{DB}$	③ reflex. prop. $\cong$
④ $\angle CBD \cong \angle ADB$	④ $\parallel \rightarrow$ alt. int. $\angle$s $\cong$
⑤ $\triangle BDC \cong \triangle DBA$	⑤ SAS $\cong$
⑥ $\overline{AB} \cong \overline{CD}$	⑥ CPCTC
⑦ ABCD is a //-gram	⑦ both pairs opp. sides $\cong \rightarrow$ //-gram

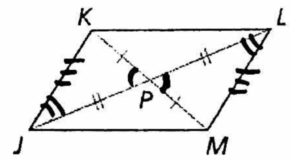
Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other,

then the quadrilateral is a parallelogram.



PROOF: **Given**: Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other
Prove: JKLM is a parallelogram



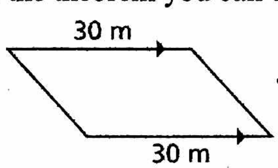
Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements	Reasons
① Diag. $\overline{JL}$, $\overline{KM}$ bisect each other @ P	① given
② $\overline{JP} \cong \overline{LP}$, $\overline{KP} \cong \overline{MP}$	② def. of seg. bisector
③ $\angle KPJ \cong \angle MPL$	③ vert. $\angle$s $\cong$
④ $\triangle KPJ \cong \triangle MPL$	④ SAS $\cong$
⑤ $\overline{KJ} \cong \overline{ML}$, $\angle KJP \cong \angle MLP$	⑤ CPCTC
⑥ $\overline{KJ} \parallel \overline{ML}$	⑥ alt. int. $\angle$s $\cong \rightarrow \parallel$
⑦ JKLM is a $\parallel$-gram	⑦ same pair opp. sides $\cong$ + $\parallel$ $\rightarrow$ $\parallel$-gram.

Example:

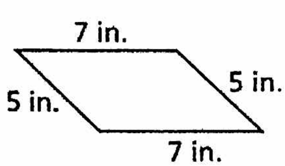
3. State the theorem you can use to show that the quadrilateral is a parallelogram.

a.



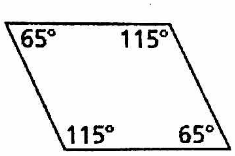
yes! same pair opp. sides $\cong$ + $\parallel$ $\rightarrow$ $\parallel$-gram

b.



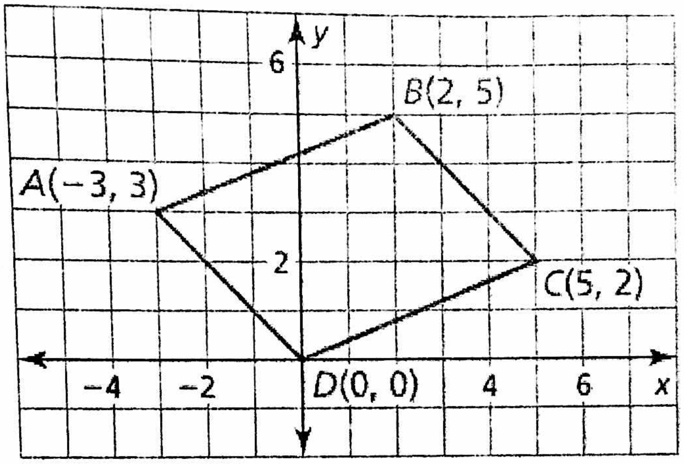
yes! both pairs opp. sides $\cong$ $\rightarrow$ $\parallel$-gram

c.



yes! both pairs opp. $\angle$s $\cong$ $\rightarrow$ $\parallel$-gram

4. Show that quadrilateral ABCD is a parallelogram.



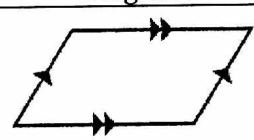
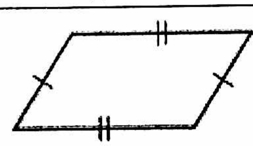
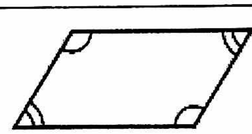
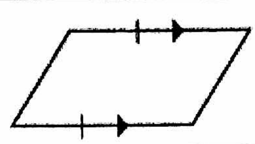
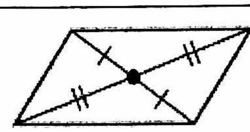
mdpt $\overline{AC}$ = mdpt $\overline{BD}$

$\left(\frac{-3+5}{2}, \frac{3+2}{2}\right) = \left(\frac{0+2}{2}, \frac{5+0}{2}\right)$

$\left(\frac{2}{2}, \frac{5}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right)$

$(1, 2.5) = (1, 2.5)$

since the mdpts are the same for either diag, then the diags. bisect each other. ∴ ABCD is a ∥-gram.

Ways to Prove a Quadrilateral is a Parallelogram		
Theorem or Definition	*Diagram*	*Method for Coordinate Plane*
Show that both pairs of opposite sides are parallel.		show slopes of opp. sides are = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$
Show that both pairs of opposite sides are congruent.		dist. formula to show opp. sides are =
Show that both pairs of opposite angles are congruent.		
Show that one pair of opposite sides are congruent and parallel.		dist. formula + slope formula on same set of opp sides
Show that the diagonals bisect each other. ☆☆		show mdpts of both diags. are same